

Power-counting renormalizability of generalized Hořava gravity

Matt Visser

School of Mathematics, Statistics, and Operations Research, Victoria University of Wellington, New Zealand

(Dated: 24 December 2009; L^AT_EX-ed December 24, 2009)

In an earlier article [arXiv: 0902.0590 [hep-th], Phys. Rev. **D80** (2009) 025011], I discussed the potential benefits of allowing Lorentz symmetry breaking in quantum field theories. In particular I discussed the perturbative power-counting *finiteness* of the normal-ordered $:P(\phi)_{d+1}^{z \geq d}:$ scalar quantum field theories, and sketched the implications for Hořava’s model of quantum gravity. In the current rather brief addendum, I will tidy up some dangling issues and fill out some of the technical details of the argument indicating the power-counting *renormalizability* of a $z \geq d$ variant of Hořava gravity in $(d+1)$ dimensions.

PACS numbers: 11.30.Cp 03.70.+k 11.10.Kk 11.25.Db 04.60.-m

Keywords: Lorentz symmetry; regularization; renormalization; finite QFTs; Hořava gravity

I. INTRODUCTION

In reference [1] I argued for the perturbative power-counting *finiteness* of the normal-ordered $:P(\phi)_{d+1}^{z=d}:$ scalar quantum field theories in $(d+1)$ dimensions, where the central defining feature of these quantum field theories is that the kinetic term in the Lagrangian contains exactly two time derivatives and up to $2d = 2z$ space derivatives.

I then rather briefly sketched the implications of this result for the perturbative power-counting *renormalizability* of a suitably defined $(d+1)$ dimensional version of Hořava gravity, where the central defining feature of this model is again the presence of exactly two time derivatives and up to $2d = 2z$ space derivatives in the Lagrangian. This is a natural generalization of the specific $z = 3$ modification of $(3+1)$ gravity that was explicitly introduced by Hořava [2].

In that earlier article [1], I terminated the discussion at the stage where it became clear that the perturbatively quantized graviton field could consistently be assigned a canonical dimension of zero — this being the standard perturbative signal that arbitrarily high-order Feynman diagrams behave no worse than low-order Feynman diagrams, thereby *implicitly* implying (power-counting) renormalizability. However I did not include any *explicit* power-counting argument. Since this omission has led to some ongoing confusion, I will in this brief addendum to the previous article [1] tidy up the argument by providing the missing details.

In particular, I will supplement that previous discussion with a more explicit argument involving the superficial degree of divergence, and will also take advantage of this opportunity to extend the discussion to consider the situation for $z > d$. Despite the considerable activity regarding other aspects of Hořava gravity, regarding renormalizability so far very little has been done that goes beyond simple power counting arguments.

II. POWER-COUNTING

Recall that for the $P(\phi)_{d+1}^z$ scalar quantum field theories each loop integral has dimension $[\kappa]^{d+z}$, while each propagator has dimension $[\kappa]^{-2z}$. To analyze the superficial degree of divergence one need only consider the one-particle-irreducible (1PI) sub-diagrams of the Feynman diagram. For each such 1PI sub-diagram the total contribution to dimensionality coming from loop integrals and internal propagators is $[\kappa]^{(d+z)L-2Iz}$, which is summarized by saying that the “superficial degree of divergence” is

$$\delta = (d+z)L - 2Iz = (d-z)L - 2(I-L)z. \quad (1)$$

Note that the quantity I only counts the propagators internal to the 1PI sub-diagram. But to get L loops one needs, at the very least, I internal propagators. So for any 1PI Feynman diagram we certainly have

$$\delta \leq (d-z)L. \quad (2)$$

Consequently, if one picks $z \geq d$ then

$$\delta \leq 0, \quad (3)$$

and the *worst* divergence one can possibly encounter is logarithmic. (Or a power of a logarithm if one has several subgraphs with $\delta = 0$.)

Since at this stage of the argument we have already assumed $z \geq d$, such a logarithmic divergence can occur only for the borderline case $z = d$ and $L = I$, that is for a “rosette” Feynman diagram. This observation is enough to guarantee that the non-normal-ordered $P(\phi)_{d+1}^{z=d}$ is power-counting *renormalizable*, and to render the normal-ordered $:P(\phi)_{d+1}^{z=d}:$ power-counting *finite*. Furthermore if one takes $z > d$ this discussion is sufficient to render $P(\phi)_{d+1}^{z>d}$ (with or without normal ordering) power-counting *finite*.

Turning our attention now to a $z \geq d$ variant of Hořava gravity in $(d+1)$ dimensions, (containing up to $2z$ spatial derivatives of the d dimensional spatial metric), one obtains the same power-counting for the loop integrals and the propagators — *the difference now lies in the graviton self-interaction vertices*. While the vertices for the scalar field theory carried no factors of momentum, for Hořava gravity and its variants the graviton self-interaction vertices arise from a perturbative action of the form [1]

$$S \sim \int \left\{ \dot{h}^2 + P(\nabla^{2z}, h) \right\} dt d^d x, \quad (4)$$

where $P(\nabla^{2z}, h)$ is now an infinite-order polynomial in the graviton field h , which contains up to $2z$ spatial derivatives.

In contrast to the scalar self-interaction vertices, the graviton self-interaction vertices thus contain up to $2z$ factors of momentum. If these are external momenta they do not contribute to the superficial degree of divergence. However internal momenta, and for any 1PI Feynman diagram with V vertices there can be up to $2zV$ factors of internal momenta, do contribute to the superficial degree of divergence. Consequently we now have the inequality

$$\delta \leq (d+z)L + 2z(V-I) = (d-z)L + 2z(V+L-I). \quad (5)$$

But as always, Euler's theorem for graphs implies

$$V + L - I = 1 \quad (6)$$

so that

$$\delta \leq (d-z)L + 2z. \quad (7)$$

For $z \geq d$ one simply has

$$\delta \leq 2z. \quad (8)$$

Thus the superficial degree of divergence of the 1PI Feynman diagrams is bounded by the canonical dimension of the operators already explicitly included in the bare action. This is the standard signal for renormalizability. As always, one should include in the bare action all terms compatible with the power counting and the symmetries of the theory. But that is exactly what the $z \geq d$ variant of Hořava gravity in $(d+1)$ dimensions is designed to do, and we conclude that any $z \geq d$ variant of Hořava gravity in $(d+1)$ dimensions is power-counting renormalizable. Note that instead of stopping at $z = d$ as is usually done, the present argument applies to all $z \geq d$.

III. DISCUSSION

In the specific case of $(3+1)$ dimensions, the minimal condition to get renormalizability is $z = 3$. This is the situation that is most commonly considered. The situation where “detailed balance” is invoked to suppress some terms is most clearly and forcefully introduced in [2], with additional detail provided in [3, 4]. In contrast, if one abandons detailed balance then one should include all possible terms up to $z = 3$, as has forcefully been advocated in [5, 6].

If one wishes to go beyond power-counting for Hořava gravity then the renormalizability arguments are as yet woefully incomplete. In the absence of gravity, some very useful indicative results are those of Anselmi and Halat [7], and Anselmi [8, 9, 10, 11], where the perturbative renormalizability of $z = d$ scalar-fermion-Yang-Mills field theories in flat spacetime have been investigated in considerable detail. Some progress using stochastic quantization (but limited to situations in which detailed balance applies) is reported by Orlando and Reffert in reference [12]. Extensions of this idea are reported in reference [13]. (See also Shu and Wu in reference [14].) Explicit renormalization group calculations are reported by Iengo, Russo, and Serone in reference [15]. (See also Collins *et al.* [16] for a more general analysis indicating the generic necessity for fine tuning in Lorentz violating theories.) More recently, Alexandre *et al.* [17] have used non-perturbative Schwinger-Dyson techniques to investigate $z = 3$ (fermion+scalar) Yukawa field theories in flat $(3+1)$ dimensional spacetime. In counterpoint, in the original articles [2, 3, 4] Hořava several times referred to his model as “arguably finite” — to the best of my knowledge no significant progress along those lines has been made. Because the current argument is limited to power counting, it has nothing specific to say, pro or con, regarding the vexatious issue of the scalar graviton.

Acknowledgments:

I wish to thank Jianwei Mei for some penetrating questions that encouraged me to revisit this question in more detail. This research was supported by the Marsden Fund administered by the Royal Society of New Zealand. Parts of this addendum were inspired by numerous discussions at the “Emergent Gravity IV” conference.

-
- [1] M. Visser, *Lorentz symmetry breaking as a quantum field theory regulator*, Phys. Rev. D **80** (2009) 025011 [arXiv:0902.0590 [hep-th]].
 [2] P. Hořava, *Quantum Gravity at a Lifshitz Point*, Phys. Rev. D **79**, 084008 (2009) [arXiv:0901.3775 [hep-th]].

- [3] P. Hořava, *Membranes at Quantum Criticality*, JHEP **0903** (2009) 020 [arXiv:0812.4287 [hep-th]].
 [4] P. Hořava, *Spectral Dimension of the Universe in Quantum Gravity at a Lifshitz Point*, Phys. Rev. Lett. **102** (2009) 161301 [arXiv:0902.3657 [hep-th]].

- [5] T. P. Sotiriou, M. Visser and S. Weinfurtner, *Phenomenologically viable Lorentz-violating quantum gravity*, Phys. Rev. Lett. **102** (2009) 251601 [arXiv:0904.4464 [hep-th]].
- [6] T. P. Sotiriou, M. Visser and S. Weinfurtner, *Quantum gravity without Lorentz invariance*, arXiv:0905.2798 [hep-th]. JHEP, **0910** (2009) 033.
- [7] D. Anselmi and M. Halat, *Renormalization of Lorentz violating theories*, Phys. Rev. D **76** (2007) 125011 [arXiv:0707.2480 [hep-th]].
- [8] D. Anselmi, *Weighted scale invariant quantum field theories*, JHEP **0802** (2008) 051 [arXiv:0801.1216 [hep-th]].
- [9] D. Anselmi, *Weighted power counting and Lorentz violating gauge theories. I: General properties*, Annals Phys. **324** (2009) 874 [arXiv:0808.3470 [hep-th]].
- [10] D. Anselmi, *Weighted power counting and Lorentz violating gauge theories. II: Classification*, Annals Phys. **324** (2009) 1058 [arXiv:0808.3474 [hep-th]].
- [11] D. Anselmi, *Weighted power counting, neutrino masses and Lorentz violating extensions of the Standard Model*, Phys. Rev. D **79** (2009) 025017 [arXiv:0808.3475 [hep-ph]].
- [12] D. Orlando and S. Reffert, *On the Renormalizability of Hořava-Lifshitz-type Gravities*, Class. Quant. Grav. **26** (2009) 155021 [arXiv:0905.0301 [hep-th]].
- [13] D. Orlando and S. Reffert, *On the Perturbative Expansion around a Lifshitz Point*, arXiv:0908.4429 [hep-th].
- [14] F. W. Shu and Y. S. Wu, *Stochastic Quantization of the Hořava Gravity*, arXiv:0906.1645 [hep-th].
- [15] R. Iengo, J. G. Russo and M. Serone, *Renormalization group in Lifshitz-type theories*, JHEP **0911** (2009) 020 [arXiv:0906.3477 [hep-th]].
- [16] J. Collins, A. Perez, D. Sudarsky, L. Urrutia and H. Vucetich, *Lorentz invariance and quantum gravity: an additional fine-tuning problem?*, Phys. Rev. Lett. **93** (2004) 191301 [arXiv:gr-qc/0403053].
- [17] J. Alexandre, K. Farakos, P. Pasipoularides and A. Tsapalis, *Dynamical generation of Lorentz symmetry for a Lifshitz-type Yukawa model*, arXiv:0909.3719 [hep-th].